Technical Comments

Comment on "Transient Surface Temperatures in Rocket Nozzles"

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Nomenclature

a = cylinder inside radius
b = cylinder outside radius
c_p = specific heat
d = plate thickness, cylinder thickness
h = heat-transfer coefficient
J = Bessel function of the first kind
k = thermal conductivity
N_a = Biot number, ha/k
r = cylinder radius, $a \leq r \leq b$
t = time
T = temperature
Y = Bessel function of the second kind
θ = $(T - T_g)/(T_g - T_i)$
μ_n = eigenvalues defined by Eq. (4)
ρ = density
τ_a = $kt/\rho c_p a^2$
Ω = b/a
ω = r/a

Subscripts

g = gas
i = initial
0 = zero order
1 = first order

In their recent note, Chao, Jacobsen, and Anderson[1] state that "to calculate the transient temperature response of the inside surface of the rocket nozzle, one can use either a simple analytic solution limited to the flat plate geometry . . ." They did not mention that solutions (e.g., Ref. 2) based on hollow cylinders are also available. Concerned that others engaged in rocket nozzle heat-transfer work and nozzle design may be unaware of the transient radial heat-conduction solution in a homogeneous hollow cylinder, the present authors are presenting the solution as follows:

$$\theta(\omega, \tau_a) = \sum_{n=1}^{\infty} A_n R_0(\mu_n\omega) \exp(-\mu_n{}^2\tau_a) \tag{1}$$

where

$$A_n = -\frac{2N_a R_0(\mu_n)}{(\mu_n\Omega)^2 R_0{}^2(\mu_n\Omega) - (\mu_n{}^2 + N_a{}^2)R_0{}^2(\mu_n)}$$

and

$$R_0(\mu_n\omega) = \frac{J_0(\mu_n\omega)}{J_1(\mu_n\Omega)} - \frac{Y_0(\mu_n\omega)}{Y_1(\mu_n\Omega)}$$

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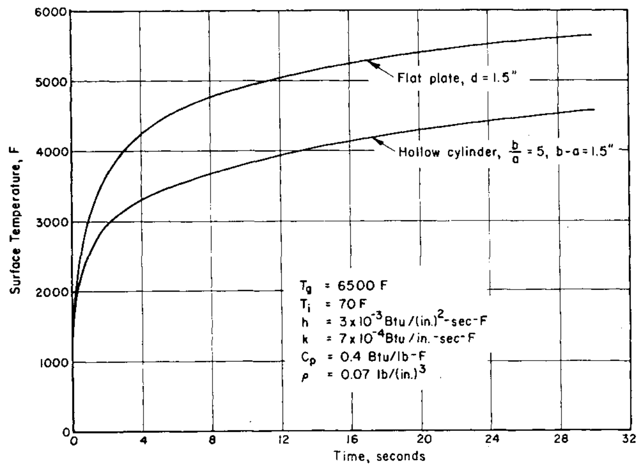

Fig. 1 Comparison of hollow-cylinder and flat-plate surface temperature histories.

The difference between Eq. (1) and Mayer's solution[2] is that a uniform initial cylinder temperature other than zero is taken into account in the preceding solution. The boundary conditions used to obtain Eq. (1) are

$$\left.\begin{array}{lll} \text{at } r = a & \omega = 1 & \partial\theta/\partial\omega = (ha/k)\theta \\ \text{at } r = b & \omega = b/a & \partial\theta/\partial\omega = 0 \end{array}\right\} \tag{2}$$

The initial conditions are

$$\text{at } \tau = 0 \qquad \theta(\omega, 0) = -1 \tag{3}$$

The eigenvalues μ_n are determined by solving the equation

$$-\mu_n[R_1(\mu_n)/R_0(\mu_n)] = N_a \tag{4}$$

where

$$R_1(\mu_n) = \frac{J_1(\mu_n)}{J_1(\mu_n\Omega)} - \frac{Y_1(\mu_n)}{Y_1(\mu_n\Omega)}$$

For the temperature response of the gas-side surface of the nozzle, the only substitution necessary in Eq. (1) is to let $\omega = 1$.

Figure 1 shows a comparison of the gas-side surface temperature histories for a hollow cylinder and a flat plate of equal wall thickness. The values of the material thermal and physical properties used are typical of graphite. For the particular set of values chosen, a maximum error in temperature of approximately 30% results from using a flat-plate geometry as an approximation of a hollow cylinder. In general, use of the flat plate will lead to conservative design estimates since the predicted temperatures are higher than would occur in a hollow cylinder. For very large values of Ω (the ratio of outside radius to inside radius), the error in surface temperature history associated with the flat-plate assumption may be larger than those errors inherent in using constant material thermal properties and heat-transfer coefficient.

References

[1] Chao, G. T. Y., Jacobsen, J. A., and Anderson, J. T., "Transient surface temperatures in rocket nozzles," J. Spacecraft Rockets 1, 219–222 (1964).

[2] Mayer, E., "Analysis of temperature transients in rocket walls," M. W. Kellog Rept. SPD 169 (1948).